

Fractional Quantum Hall Effect and Wigner Crystal of Two-Flux Composite Fermions

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In two-dimensional electron systems confined to GaAs quantum wells, as a function of either tilting the sample in magnetic field or increasing density, we observe multiple transitions of the fractional quantum Hall states (FQHSs) near filling factors $\nu = 3/4$ and $5/4$. The data reveal that these are spin-polarization transitions of interacting two-flux composite Fermions, which form their own FQHSs at these fillings. The fact that the reentrant integer quantum Hall effect near $\nu = 4/5$ always develops following the transition to full spin polarization of the $\nu = 4/5$ FQHS strongly links the reentrant phase to a pinned *ferromagnetic* Wigner crystal of two-flux composite Fermions.

Fractional quantum Hall states (FQHSs) are among the most fundamental hallmarks of ultra-clean interacting two-dimensional electron systems (2DESs) at a large perpendicular magnetic field (B_{\perp}) [1]. These incompressible quantum liquid phases, signaled by the vanishing of the longitudinal resistance (R_{xx}) and the quantization of the Hall resistance (R_{xy}), can be explained by mapping the interacting electrons to a system of essentially non-interacting, $2p$ -flux composite Fermions (2^p CFs), each formed by attaching $2p$ magnetic flux quanta to an electron (p is an integer). The 2^p CFs have discrete energy levels, the so-called Λ -levels, and the FQHSs of electrons seen around Landau level (LL) filling factor $\nu = 1/2$ ($1/4$) would correspond to the integer quantum Hall states of 2^p CFs (4^p CFs) at integral ν^{CF} [2]. In state-of-the-art, high-mobility 2DESs, FQHSs also develop around $\nu = 3/4$, and are usually understood as the particle-hole counterparts of the FQHSs near $\nu = 1/4$ through the relation $\nu \leftrightarrow (1 - \nu)$ [3] [4]. Alternatively, these states might also be the FQHSs of *interacting* 2^p CFs at $\nu^{CF} = \nu/(1 - 2\nu)$. For example, the $\nu = 4/5$ state is the $\nu^{CF} = -4/3$ FQHS of 2^p CFs, and has the same origin as the unconventional FQHS seen at $\nu = 4/11$ ($\nu^{CF} = 4/3$) [5, 6].

Another hallmark of clean 2DESs is an insulating phase that terminates the series of FQHSs at low fillings, near $\nu = 1/5$, [7, 8]. This insulating phase is generally believed to be an electron Wigner crystal, pinned by the small but ubiquitous disorder potential [9]. Recently, an insulating phase was observed near $\nu = 4/5$ in clean 2DESs [10, 11]. This phase, which is signaled by a reentrant integer quantum Hall state (RIQHS) near $\nu = 1$, was interpreted as the particle-hole symmetric state of the Wigner crystal seen at very small ν [10, 11]. In this picture, the holes, unoccupied states in the lowest LL, have filling factor $\nu^h \sim 1/5$ ($= 1 - 4/5$) and form a liquid phase when the short-range interaction is strong; see the left panel of Fig. 1(a). They turn into a solid phase when the thickness of the 2DES increases and the long-range interaction dominates (right panel of Fig. 1(a)). This interpretation is plausible, since the RIQHS only appears when the well-width (W) is more than five times larger than the magnetic length. However, it does not predict or allow for any transitions of the $\nu = 4/5$ FQHS, which

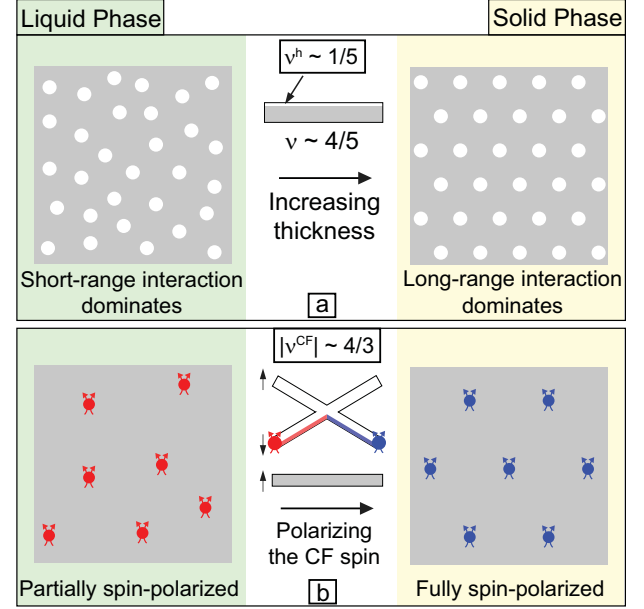


FIG. 1. Schematic figures illustrating two different explanations of the RIQHS near $\nu = 4/5$. (a) The hole Wigner crystal picture. The electrons at $\nu \sim 4/5$ are equivalent to holes at $\nu^h \sim 1/5$. These holes condense into a liquid phase when the short-range interaction dominates (left), and into a crystal phase when the long-range interaction dominates in thicker 2DESs (right). The gray background represents electrons, and white circles represent the holes. (b) The 2^p CF Wigner crystal picture. The electrons at $\nu \sim 4/5$ are equivalent to 2^p CFs at $|\nu^{CF}| \sim 4/3$. There is one fully-filled, spin-up Λ -level (the gray background), and the rest of the 2^p CFs can either be spin-down (red) and form a liquid phase when the Zeeman energy (E_Z) is small (left panel), or be spin-up (blue) at large E_Z and form a ferromagnetic crystal phase of 2^p CFs (right panel).

is always seen just before the RIQHS develops [10].

Here we report our extensive study of the FQHSs near $\nu = 3/4$ (at $4/5$ and $5/7$) and their particle-hole counterparts near $\nu = 5/4$ (at $6/5$ and $9/7$) [3]. Via either increasing the 2DES density or the tilt angle between the magnetic field and the sample normal, we increase the ratio of the Zeeman energy ($E_Z = |g|\mu_B B$ where B is the total magnetic field) to Coulomb en-

ergy ($V_C = e^2/4\pi\epsilon l_B$ where $l_B = \sqrt{\hbar/eB_\perp}$ is the magnetic length), and demonstrate that these FQHSs undergo multiple transitions as they become spin polarized. The number of observed transitions, one for the FQHSs at $\nu = 4/5$ and $6/5$, and two for the states at $\nu = 5/7$ and $9/7$, is consistent with what is expected for polarizing the spins of *interacting* ^2CF s. Note that these interacting ^2CF s form FQHSs at *fractional* CF fillings $\nu^{CF} = -4/3$ ($\nu = 4/5$ and $6/5$) and $-5/3$ ($\nu = 5/7$ and $9/7$). Even more revealing is the observation that, whenever the RIQHS near $\nu = 4/5$ develops, it is preceded by a transition of the FQHS at $\nu = 4/5$ to a fully spin-polarized ^2CF state. This provides evidence that the RIQHS is the manifestation of a *ferromagnetic* ^2CF Wigner crystal (see Fig. 1(b)).

We studied 2DESs confined to wide GaAs quantum wells (QWs) bounded on each side by undoped $\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$ spacer layers and Si δ -doped layers, grown by molecular beam epitaxy. We report here data for two samples, with QW widths $W = 65$ and 60 nm, and as-grown densities of $n \simeq 1.4$ and 0.4 , in units of 10^{11} cm^{-2} which we use throughout this report. The samples have a van der Pauw geometry with InSn contacts at their corners, and each is fitted with an evaporated Ti/Au front-gate and an In back-gate. We carefully control n while keeping the charge distribution symmetric. The measurements were carried out in dilution refrigerators with a base temperature of $T \simeq 25$ mK and superconducting magnets. We used low-frequency ($\lesssim 40$ Hz) lock-in techniques to measure the transport coefficients.

Figure 2(a) shows R_{xx} and R_{xy} magnetoresistance traces near $\nu = 3/4$ measured in a symmetric 65-nm-wide QW, at densities ranging from 1.00 to 1.54. The deep R_{xx} minimum seen at $\nu = 4/5$ in the lowest density ($n = 1.00$) trace disappears at $n = 1.13$ and reappears at higher densities. With increasing n , an R_{xx} minimum also develops to the left of $\nu = 4/5$, and merges with the $\nu = 1$ $R_{xx} = 0$ plateau at the highest density $n = 1.54$ (see down arrows in Fig. 2(a)). Meanwhile, two minima appear in R_{xy} on the sides of $\nu = 4/5$ when the $\nu = 4/5$ FQHS reappears (up-arrows in Fig. 2(a)). These two R_{xy} minima become deeper at higher densities and, at $n \simeq 1.54$, the R_{xy} minimum on the left side of $\nu = 4/5$ merges into the $\nu = 1$ $R_{xy} = h/e^2$ plateau [12].

The R_{xx} and R_{xy} data of Fig. 2(a) provide evidence for the development of a RIQHS between $\nu = 4/5$ and 1, as reported recently and attributed to the formation of a pinned Wigner crystal state [10, 11]. Note that at the onset of this development, the $\nu = 4/5$ FQHS shows a transition manifested by a weakening and strengthening of its R_{xx} minimum. The central questions we address here are: What is the source of this transition, and what does that imply for the origin of the $\nu = 4/5$ FQHS and the nearby RIQHS?

As Fig. 2(b) illustrates, the R_{xx} and R_{xy} traces measured in the same QW at a fixed density $n = 1.00$ and

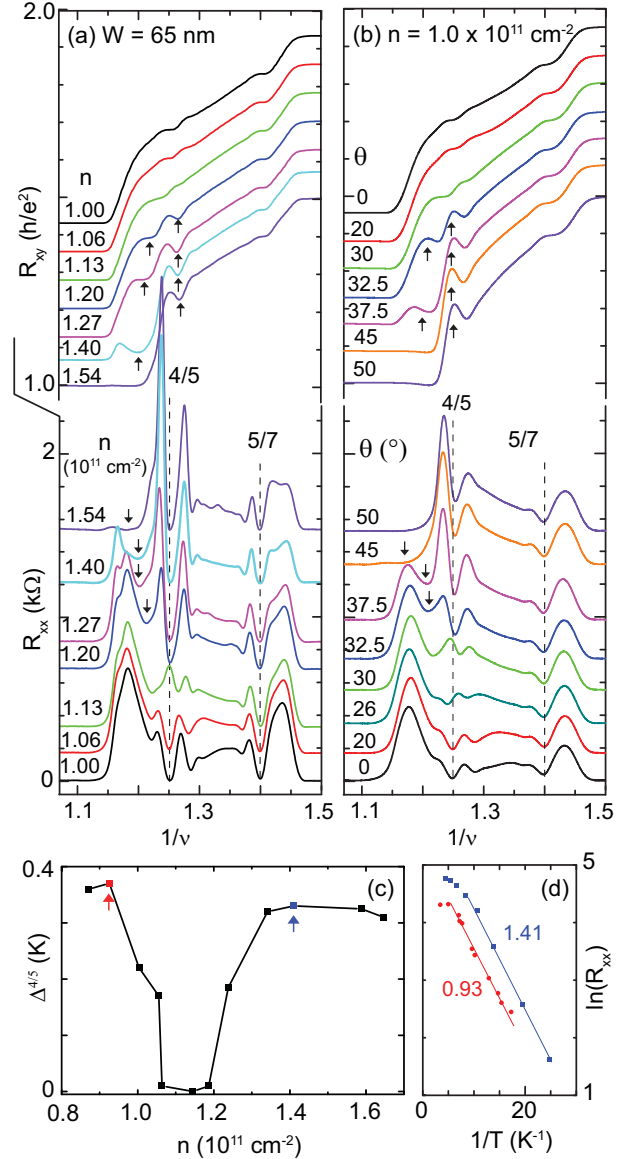


FIG. 2. Longitudinal (R_{xx}) and Hall (R_{xy}) magnetoresistance traces for 2D electrons confined to a 65-nm-wide GaAs QW near $\nu = 3/4$ as a function of (a) increasing charge density, and (b) tilting the sample in the magnetic field. The density n (in units of 10^{11} cm^{-2}) or tilting angle θ for each trace is indicated, and traces are shifted vertically for clarity. (c) Energy gap of the $\nu = 4/5$ FQHS as a function of n . (d) Arrhenius plot of R_{xx} vs. $1/T$ at $n = 0.93$ and $1.41 \times 10^{11} \text{ cm}^{-2}$.

different tilting angles θ reveal an evolution very similar to the one seen in Fig. 2(a) (θ denotes the angle between the magnetic field and normal to the 2D plane). At $\theta = 0^\circ$, a strong FQHS is seen at $\nu = 4/5$. The R_{xx} minimum at $\nu = 4/5$ disappears at $\theta \simeq 30^\circ$ and reappears at higher θ , signaling the destruction and resurrection of the FQHS. Two minima in R_{xy} on the sides of $\nu = 4/5$, marked by the up-arrows, develop at $\theta > 30^\circ$. As θ is further increased, the R_{xy} minimum to the left

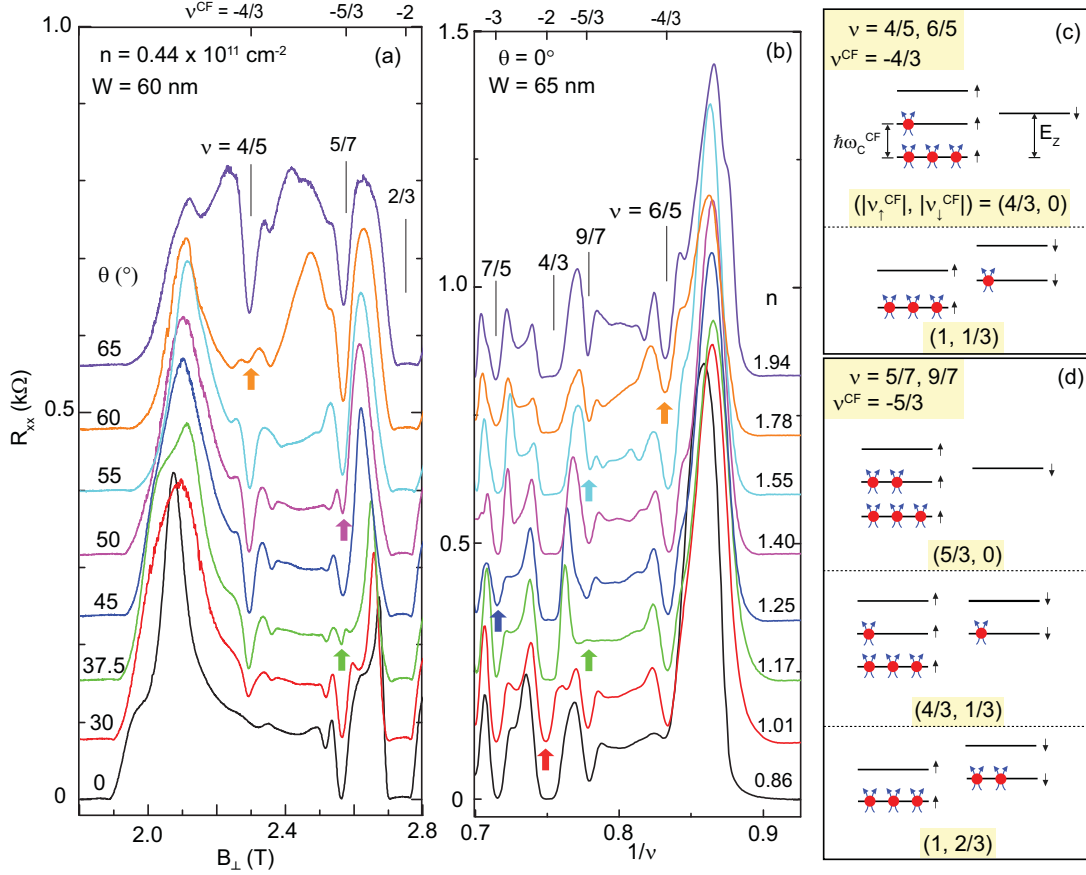


FIG. 3. (a) R_{xx} measured in a 60-nm-wide QW near $\nu = 3/4$, at a fixed density $n = 0.44 \times 10^{11} \text{ cm}^{-2}$ and different tilting angles θ . (b) R_{xx} for a 65-nm-wide QW near $\nu = 5/4$, at $\theta = 0^\circ$ and different densities $n = 0.86$ to $1.94 \times 10^{11} \text{ cm}^{-2}$. (c, d) Schematic plots showing multiple configurations of the $\nu = 4/5$ and $6/5$, and $5/7$ and $9/7$ FQHSs with different spin-polarizations.

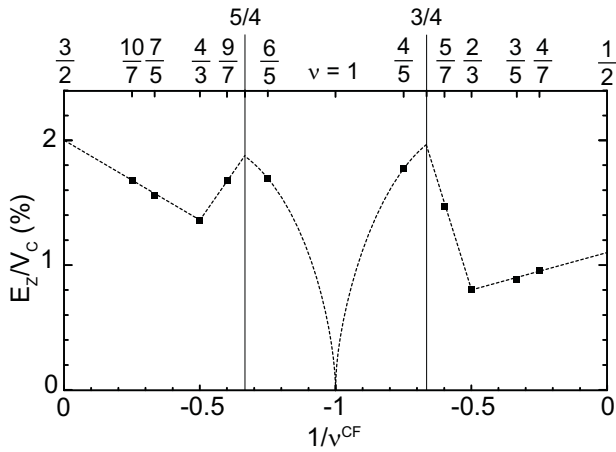


FIG. 4. Summary of the spin-polarization energy in units of the Coulomb energy, E_Z/V_C , at different filling factors (ν). Dotted lines are guides to the eye. All data points were measured in symmetric QWs. The transitions at $\nu = 2/3$, $3/5$ and $4/7$ were measured in perpendicular magnetic field by changing n , and the rest at a fixed density $n = 0.44$ by changing θ . For each filling, only the (last) transition into a fully spin-polarized configuration is shown.

of $\nu = 4/5$ deepens and an R_{xx} minimum starts to appear at the same ν (see down-arrows in Fig. 2(b)). At the highest tilting angles $\theta > 40^\circ$, these minima merge into the $\nu = 1$ R_{xy} Hall plateau quantized at h/e^2 , and the $R_{xx} = 0$ minimum near $\nu = 1$ plateau, respectively. This evolution with increasing θ is clearly very similar to what is seen as a function of increasing electron density. Moreover, it suggests that the $\nu = 4/5$ FQHS transition is induced by the enhancement of E_Z and is therefore spin related, similar to the spin-polarization transitions observed for other FQHSs [13–16].

Observing a spin-polarization transition for the $\nu = 4/5$ FQHS, however, is surprising as this state is usually interpreted as the particle-hole counterpart of the $\nu = 1/5$ FQHS, which is formed by *non-interacting* four-flux ^4CFs . Such a state should be always fully spin-polarized and no spin-polarization transition is expected [4]. On the other hand, if the $\nu = 4/5$ FQHS is interpreted as the FQHS of *interacting* two-flux CFs (^2CFs), then it corresponds to the $\nu^{CF} = -4/3$ FQHS of ^2CFs , and has two possible spin configurations as shown in Fig. 3(c). The system has one fully-occupied, spin-up, Λ -level and

one $1/3$ -occupied Λ -level. Depending on whether E_Z is smaller or larger than the Λ -level separation of the 2CFs , the $1/3$ -filled Λ -level may be either spin-down or spin-up (see Fig. 3(c)) [17].

To further test the validity of the above interpretation, we measured R_{xx} in a 60-nm-wide QW at a very low density $n = 0.44$ and different θ in Fig. 3(a). The $\nu = 4/5$ FQHS exhibits a clear transition at $\theta = 60^\circ$, manifested by a weakening of the R_{xx} minimum. Note that the transition of the $\nu = 4/5$ FQHS appears in Figs. 2(a), 2(b) and 3(a) when the ratio of the Zeeman to Coulomb energies (E_Z/V_C) is about 0.0145, 0.0157 and 0.0177, respectively. The electron layer-thicknesses at these three transitions, parameterized by the standard deviation (λ) of the charge distribution in units of l_B , are 1.66, 1.52 and 0.75, respectively. The softening of the Coulomb interaction due to the finite-layer-thickness effect is less and the spin-polarization energy should be higher for smaller λ/l_B (see [16] for the dependence of spin-polarization energy on the finite-layer-thickness). Therefore, these values are consistent with each other.

In Fig. 3(a), we also observe two transitions for the $\nu = 5/7$ FQHS at $\theta = 37.5^\circ$ and 50° , suggesting three different phases. This observation is consistent with the $\nu = 5/7$ FQHS being formed by interacting 2CFs . In such a picture, the $\nu = 5/7$ FQHS, which is the $\nu^{CF} = -5/3$ FQHS of the 2CFs , has three different possible spin configurations, as shown in Fig. 3(d). Similar to Fig. 3(c), the lowest spin-up Λ -level is always fully occupied. The second Λ -level is $2/3$ -occupied spin-up (spin-down), if E_Z is larger (smaller) than the Λ -level separation. If E_Z equals the Λ -level separation, the 2CFs form a novel spin-singlet state when the spin-up and spin-down Λ -levels are both $1/3$ -occupied; see the middle panel of Fig. 3(d).

Data near $\nu = 5/4$ measured in the 65-nm-wide QW at different densities, shown in Fig. 3(b), further confirm our picture. The $\nu = 6/5$ and $9/7$ FQHSs exhibit transitions similar to their particle-hole conjugate states at $\nu = 4/5$ and $5/7$, respectively. The $\nu = 6/5$ FQHS shows a transition at $n = 1.78$. At this transition, $E_Z/V_C \simeq 0.0149$ and $\lambda/l_B \simeq 1.86$, very similar to the corresponding values (0.0145 and 1.66) at $\nu = 4/5$ in Fig. 2(a), suggesting that the particle-hole symmetry $\nu \leftrightarrow (2 - \nu)$ is conserved in this case [18]. Furthermore, the $\nu = 9/7$ FQHS becomes weak twice, at $n = 1.17$ and 1.55, also consistent with the $\nu = 5/7$ FQHS transitions.

It is instructive to compare the transitions we observe at fractional ν^{CF} with the spin-polarization transitions of other FQHSs at integer ν^{CF} . In Fig. 4, we summarize the critical E_Z/V_C above which the FQHSs between $\nu = 1/2$ and $3/2$ become fully spin-polarized. The measurements were all made on the 60-nm-wide QW. The x -axis is $1/\nu^{CF}$, and we mark the electron LL filling factor ν in the top axis. The dotted lines, drawn as guides to the eye, represent the phase boundary between fully spin-polarized (above) and partially spin-polarized (be-

low) 2CFs . Note that the system is always fully spin-polarized at $\nu^{CF} = -1$ ($\nu = 1$) [19, 20]. The critical E_Z/V_C of FQHSs with integral ν^{CF} increases with ν^{CF} and reaches maxima at $\nu^{CF} = -\infty$ ($\nu = 1/2$ and $3/2$). Secondary maxima in the boundaries appear at $\nu^{CF} = -3/2$ ($\nu = 3/4$ and $5/4$), and seem to have approximately the same height as at $\nu = 1/2$ and $3/2$.

While Fig. 3 data strongly suggest that we are observing spin transitions of various FQHSs, there is also some theoretical justification. It has been proposed that the enigmatic FQHSs observed at $\nu = 4/11$ and $5/13$ in the highest quality samples can be interpreted as the FQHSs of interacting 2CFs at $\nu^{CF} = +4/3$ and $+5/3$ [5, 6]. A recent theoretical study predicts a transition of the $\nu = 4/11$ FQHS to full spin polarization when E_Z/V_C is about 0.025 [21]. Our observed transition of the $\nu = 4/5$ ($\nu^{CF} = -4/3$) FQHS appears at $E_Z/V_C \simeq 0.015$ to 0.024, in different QWs with well width ranging from 65 to 31 nm and corresponding $\lambda/l_B \simeq 1.7$ to 1.1 [10], consistent with this theoretically predicted value.

Another useful parameter in characterizing the origin of the $\nu = 4/5$ FQHS and its transition are the energy gaps on the two sides of the transition. We show in Fig. 2(c) the measured excitation gaps for this state at different densities in the 65-nm-wide QW. The $\nu = 4/5$ FQHS transition for this sample occurs at density $n \simeq 1.1$, see Fig. 2(a). Before and after the transition, e.g. at $n = 0.86$ and 1.64, the $\nu = 4/5$ FQHS has very similar energy gaps (~ 0.35 K) although the densities are different by nearly a factor of two. Since the FQHS energy gaps at a given filling are ordinarily expected to scale with $V_C \sim \sqrt{n}$, this observation suggests that the excitation gap at $\nu = 4/5$ is reduced when the FQHS becomes fully spin polarized.

Finally we revisit the RIQHSs we observe near $\nu = 4/5$ (see, e.g., Figs. 1(a) and 1(b)). These RIQHSs were interpreted as pinned Wigner crystal states [10], and the recent microwave resonance experiments confirm this interpretation [11]. Moreover, the data in Ref. [10] as well as the data we have presented here all indicate that, whenever a transition to a RIQHS occurs, it is initiated by a transition of the $\nu = 4/5$ FQHS. As we have shown here, the transition we see for the $\nu = 4/5$ FQHS is a transition to a *fully spin polarized state of interacting 2CFs* . Combining these observations leads to a tantalizing conclusion: The fact that the RIQHS near $\nu = 4/5$ always develops following a spin polarization transition of 2CFs strongly links the RIQHS to a pinned, *ferromagnetic Wigner crystal of 2CFs* , as schematically illustrated in Fig. 1(b).

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